Quiz 17

November 4, 2016

Show all work and circle your final answer.

1. $\lim_{x\to\infty} \dfrac{x^2-3}{15x^2-3x+7} \cdot \dfrac{1/x^2}{1/x^2} = \lim_{x\to\infty} \dfrac{1-3/x^2}{15-3/x+7/x^2} = \boxed{\dfrac{1}{15}}$

OR: $\overset{LH}{=} \lim_{x\to\infty} \dfrac{2x}{30x-3} = \dfrac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \dfrac{2}{30} = \boxed{\dfrac{1}{15}}$

2. $\lim_{x\to 0^+} x\ln x = 0(-\infty)$

$= \lim_{x\to 0^+} \dfrac{\ln x}{1/x} \overset{LH}{=} \lim_{x\to 0^+} \dfrac{1/x}{-1/x^2} = \lim_{x\to 0^+} \dfrac{1}{-1/x} = \lim_{x\to 0^+} -x = \boxed{0}$

$= \dfrac{-\infty}{\infty}$, so indeterminate form

3. $\lim_{x\to 0} \dfrac{x}{\cos x} = \dfrac{0}{1} = \boxed{0}$

4. $\lim_{x\to 1^-} x^{\tan(\pi x/2)}$

$= 1^{\infty}$, so indeterminate

$= \lim_{x\to 1^-} e^{\ln\left(x^{\tan(\pi x/2)}\right)}$

$= e^{\lim_{x\to 1^-} \ln\left(x^{\tan(\pi x/2)}\right)}$

$= e^{\lim_{x\to 1^-} \tan\left(\frac{\pi x}{2}\right)\ln x}$

$= e^{\lim_{x\to 1^-} \frac{\ln x}{\cot\left(\frac{\pi x}{2}\right)}}$

(Notice: $\lim_{x\to 1^-} \dfrac{\ln x}{\cot\left(\frac{\pi x}{2}\right)} = \dfrac{0}{0}$)

$\overset{LH}{=} e^{\lim_{x\to 1^-} \frac{1/x}{-\csc^2\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}}}$

$= e^{\lim_{x\to 1^-} \frac{-2}{\pi x \csc^2\left(\frac{\pi x}{2}\right)}} = \boxed{e^{\frac{-2}{\pi}}}$

(since $\csc^2\left(\frac{\pi}{2}\right) = \frac{1}{\sin^2\left(\frac{\pi}{2}\right)} = 1$)